

Name of College - S.S. College, J-Bad

Dept - Mathematics

Topic - St. Line (Contd)

Clas - B.Sc. I (Hons + Sub)

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1. Problem - By - Dr. Shree Nalini Sharma.
Find the Co-ordinates of the point of Intersection of the line

$$\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+3}{4}$$

with the plane $2x - 2y - z = 7$

Solution → By the question.

Equation of St. Line is

$$\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+3}{4} \quad \text{--- (i)}$$

and Equation of Plane is

$$2x - 2y - z = 7 \quad \text{--- (ii)}$$

We are to find the point of intersection of the given line with the given plane.

$$\text{Say } \frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+3}{4} = k$$

$$\Rightarrow x = 2k+1, y = -3k+2, z = 4k-3$$

Putting these value of x, y, z in (ii)

$$2(2k+1) - 2(-3k+2) - (4k-3) = 7$$

$$4k + 6k - 4k + 2 - 4 + 3 = 7$$

$$6k = 6$$

$$k = 1$$

$$\therefore x = 2+1 = 3$$

$$y = -3+2 = -1$$

$$z = 4-3 = 1$$

Thus the required co-ordinates of point of intersection of given line with given plane is $(3, -1, 1)$

Problem - Find the Equation of the Plane through the line $3x-4y+5z=10$ and $2x+2y-3z=4$ and parallel to $x=2y=3z$.

Solution $\rightarrow$ Here the Given plane

$$3x-4y+5z=10 \quad (i) \quad \text{and}$$

$$2x+2y-3z=4 \quad (ii)$$

and the line

$$x=2y=3z \quad \text{--- (iii)}$$

The Equation of plane through the line of intersection of plane (i) and (ii) is

$$(3x-4y+5z-10) + k(2x+2y-3z-4) = 0 \quad \text{--- (iv)}$$

$$\Rightarrow (3+2k)x + (2k-4)y + (5-3k)z - 10-4k = 0$$

Since this is parallel to the line

$$x=2y=3z$$

$$\Rightarrow \frac{x}{1} = \frac{y}{1/2} = \frac{z}{1/3}$$

$$(3+2k) \cdot 1 + (2k-4) \cdot \frac{1}{2} + (5-3k) \cdot \frac{1}{3} = 0$$

$$\Rightarrow 3+2k + k-2 + \frac{5-3k}{3} = 0$$

$$\Rightarrow 9+6k+3k-6+5-3k=0$$

$$6k+8=0$$

$$k = -8/6 = -4/3$$

Put $K = -4/8$ in (10)

$$(3x - 4y + 5z - 10) - \frac{4}{3}(2x + 2y - 3z - 4) = 0$$

$$\Rightarrow (3x - 4y + 5z - 10) \times 3 - 4(2x + 2y - 3z - 4) = 0$$

$$9x - 12y + 15z - 30 - 8x - 8y + 12z + 16 = 0$$

$$x - 20y + 27z - 14 = 0$$

$$\Rightarrow x - 20y + 27z = 14$$

Problem - Find the Equation of the perpendicular line from the point $(2, 4, -1)$ to the line

$$\frac{x+5}{4} = \frac{y+3}{2} = \frac{(z-6)}{9}$$

Obtain also the foot of the perpendicular.

Solution $\rightarrow$

Here Equation of the line is

$$\frac{x+5}{1} = \frac{y+3}{4} = \frac{(z-6)}{9} = r$$

$$x = r - 5$$

$$y = 4r - 3$$

$$\text{and } -2 = 9r - 6$$

$$\Rightarrow 2 = 6 - 9r$$

this will be the required

foot of the perpendicular of the line
Joining it to the point $(2, 4, -1)$ be
Perpendicular to the given line.

∴ The Direction ratios of the line
Joining the points $(2, 4, -1)$ and (1) are
 $x-5-2, 4x-3-4, 6-9x+1$
 $\Rightarrow x-7, 4x-7, 7-9x$

The condition for this

$$(x-7) + 4(4-7) - 9(7-9x) = 0$$

$$\Rightarrow x-7 + 16x - 28 - 63 + 81x = 0$$

$$98x = 98$$

$$\therefore x = 1$$

∴ the required foot is

$$1-5, 4 \times 1-3, 6-9$$

$$(-4, 1, -3)$$

and Direction ratios of the perpendicular
are

$$-6, -3, -2.$$

this the required perpendicular is the line
Passing through $(2, 4, -1)$ whose direction
ratios are $6, 3, 2$

∴ Required Equation is

$$\frac{x-2}{6} = \frac{y-4}{3} = \frac{z+1}{2}$$